



# 3-D Plasma Equilibrium Reconstruction at the HSX Stellarator

## Current Status and Diagnostic Development

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## Motivation

- Investigate MHD equilibrium properties via plasma reconstruction using the V3FIT code [1], experimental data and modeled signal responses [2]
- Three dimensional plasma reconstruction is necessary in stellarators to determine finite beta effects and in tokamaks to measure effects of non-axisymmetric coils
- HSX offers test bed for plasma reconstruction and benchmarking algorithms
- Magnetic diagnostic upgrade necessary for improved reconstruction by solution reduction and increased signal-to-noise ratio
- Necessary to find most sensitive diagnostics regarding the plasma equilibrium currents to be measured
- Understand the limitations of plasma reconstruction performance

## Plasma Reconstruction

- Reconstruction is used to calculate seven free parameters, which describe plasma size, plasma current and plasma pressure profile
- VMEC is used to compute the plasma equilibrium [3]
- Plasma reconstruction is based on minimizing

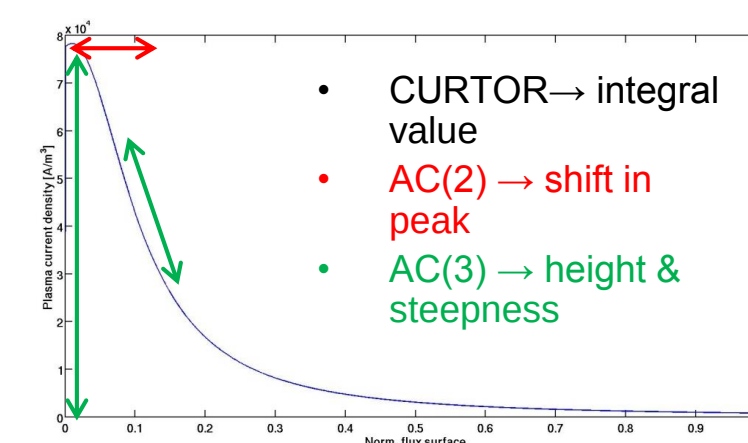
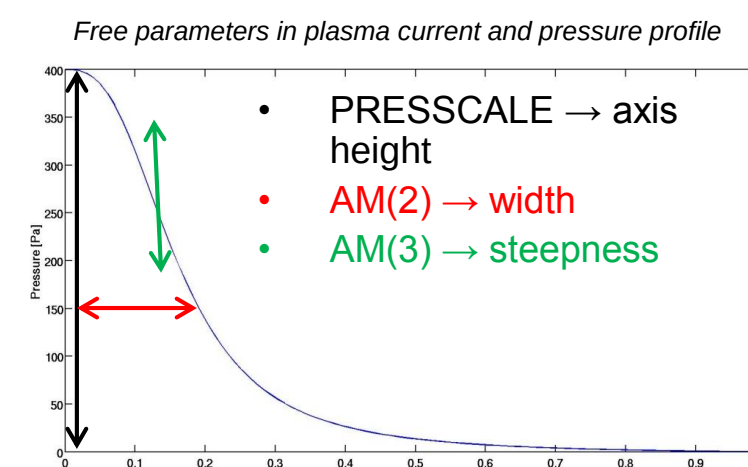
$$\chi^2 = \sum_i \left( \frac{S_i^{\text{model}}(h_j, k_j) - S_i^{\text{measure}}}{\sigma_i} \right)^2$$

deviation between measured signals  $S^{\text{measure}}$ , and predicted values  $S^{\text{model}}$ , by adjusting the model parameters

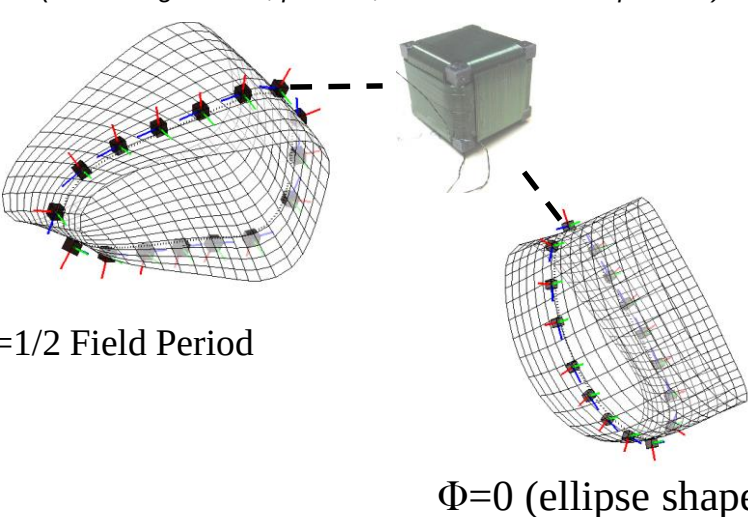
$h = \{\text{PRESSCALE}, \text{AM}(2), \text{AM}(3)\}$   
 $k = \{\text{CURTOR}, \text{AC}(2), \text{AC}(3)\}$  [1]

- Two external and one internal magnetic coil array (at two different toroidal positions) in continuous use

- By means of plasma reconstruction helical structure of Pfirsch-Schlüter current was proven at HSX [4]



Set of two external already installed magnetic diagnostic coils (measuring toroidal, poloidal, and radial field components)



## Diagnostic Positioning

### Signal Effectiveness Study (SEFFS)

- Signal effectiveness  $E_{ji}$  has been calculated for 2300 virtual diagnostic coils (local pol. & radial mag. field measurements) and a set of 1366 test equilibria (TE)

$$E_{ji} = \frac{\sigma_i^S}{\sigma_j^P} \frac{d\sigma_j^P}{d\sigma_i^S} \quad \sum_i E_{ji} = 1$$
$$= \left( \frac{1}{\sigma_i^S \sigma_j^P} (J \cdot C^P)_{ij} \right)^2$$

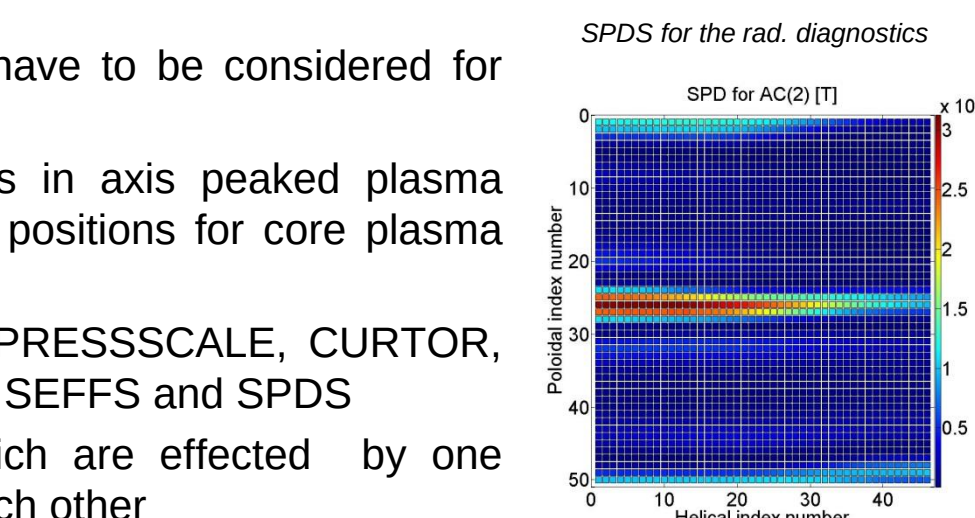
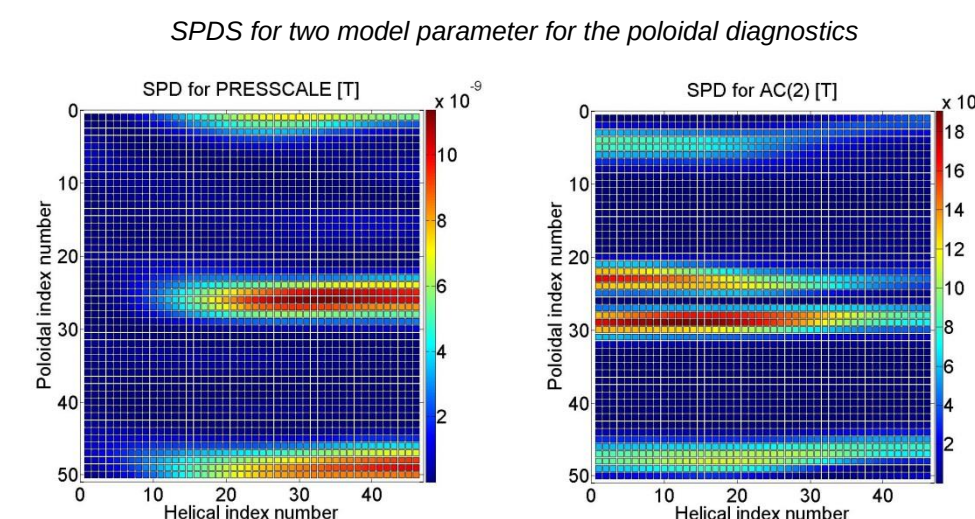
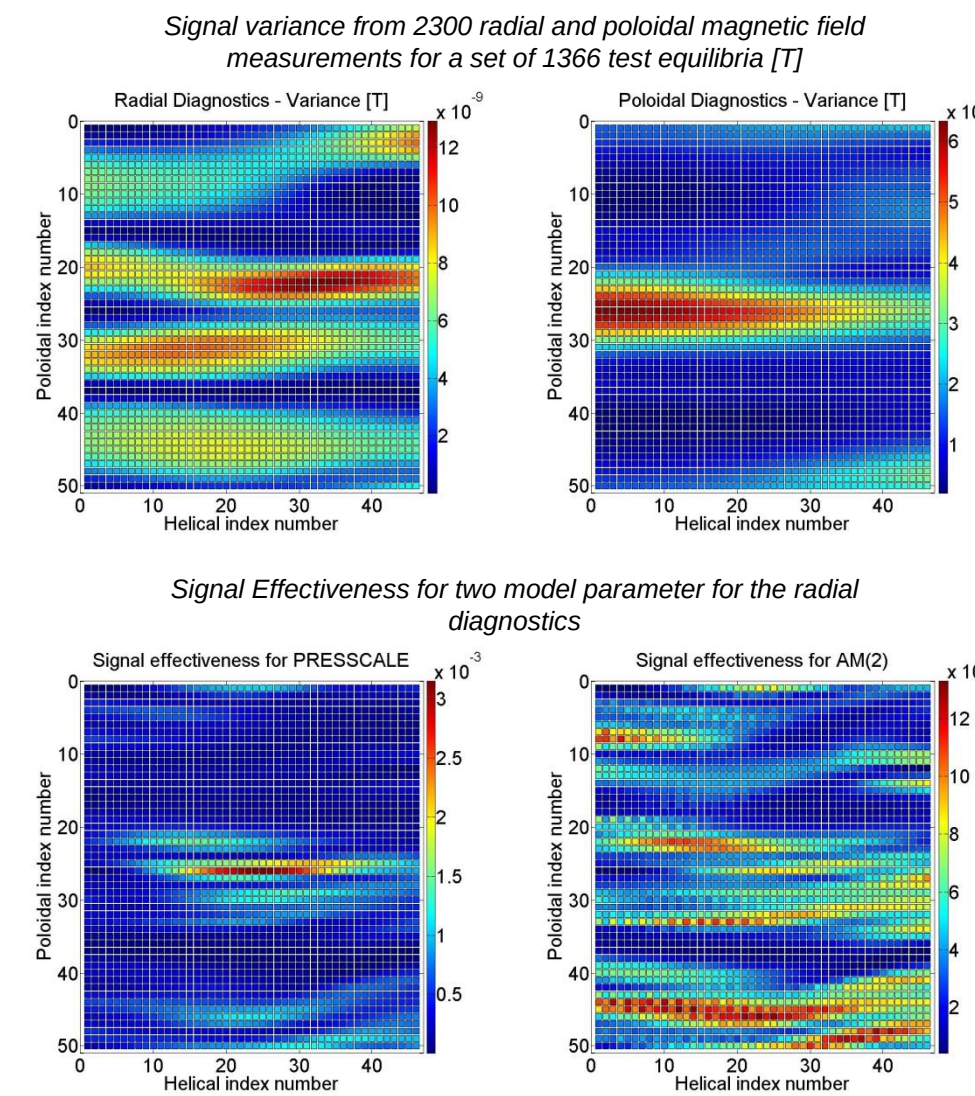
- $E_{ji}$  determines most sensitive (and thus most important) diagnostics for each model parameter
- $E_{ji}$  describes fractional reduction in j-th parameter variance  $\sigma_j^P$  for fractional reduction in i-th signal variance  $\sigma_i^S$

### Single Parameter Dependence Study (SPDS)

- Test if diagnostics with high  $E_{ji}$  undergo signal change for possible plasma scenarios at HSX
- Further set of TE have been generated where only one model parameter value was varied

$$q_{SPD,i} = \frac{\sum_{k=1}^{243} \text{var}(S_i(EQ_{k_1}), S_i(EQ_{k_2}), S_i(EQ_{k_3}))}{243}$$

- Subset of SEFFS and SPDS have to be considered for diagnostic development
- Locations sensitive to changes in axis peaked plasma currents overlap with sensitive positions for core plasma current changes
- First order parameters show (PRESSSCALE, CURTOR, show good agreement between SEFFS and SPDS
- Rad. and pol. diagnostics which are effected by one model parameter lie close to each other  
→ Construction of 25 radial and 25 poloidal magnetic field coils covering poloidal and helical index 20 – 30



## Performance Improvement

- Use of  $\chi_{\text{red}}^2$  to estimate number of reconstruction solutions

$$\chi_{\text{red}}^2 = \frac{\chi^2}{n-m-1}$$

- with  $n = \#$  of diagnostics  
and  $m = \#$  of free model parameters

- Standard reconstruction at HSX yields

$$\chi_{\text{red, recon}}^2 = 0.001 \dots 1$$

- Solution space: all equilibria which satisfy

$$\chi_{\text{red}}^2 \leq \chi_{\text{red, recon}}^2 + 1$$

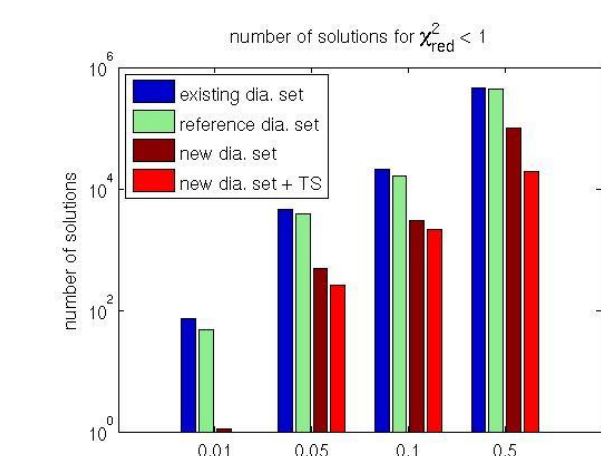
- Five dimensional scan of parameter space performed  $\approx 760,000$  equilibria
- 3 diagnostics sets

- Number of solutions reduced by factor 7 for new diagnostic set with 0.1 G signal noise

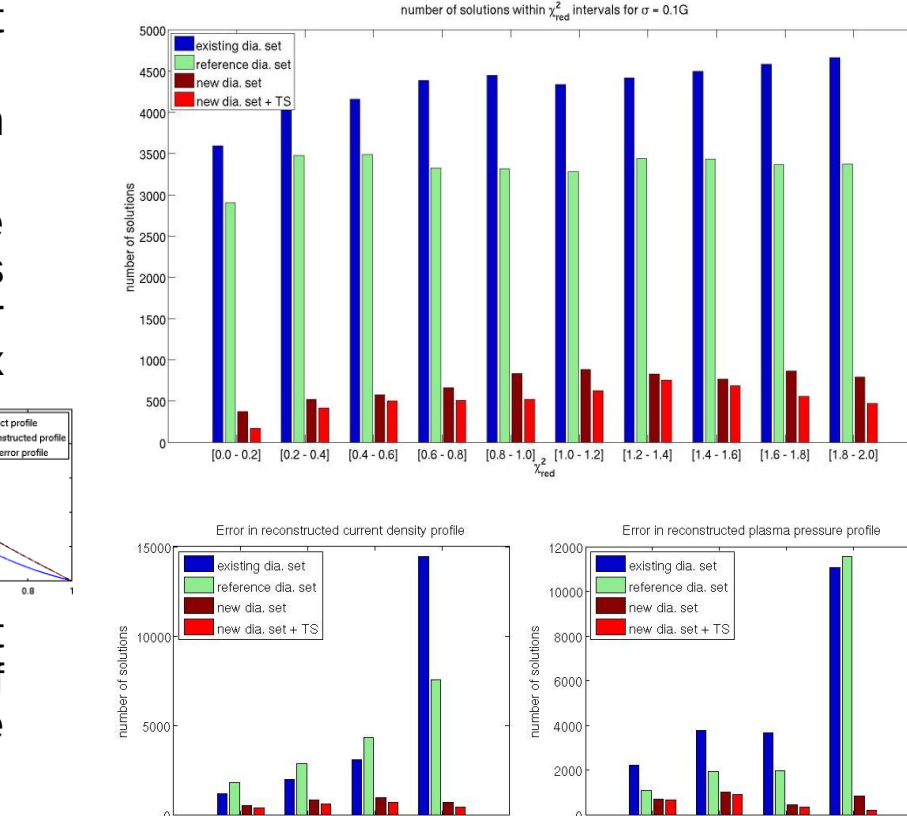
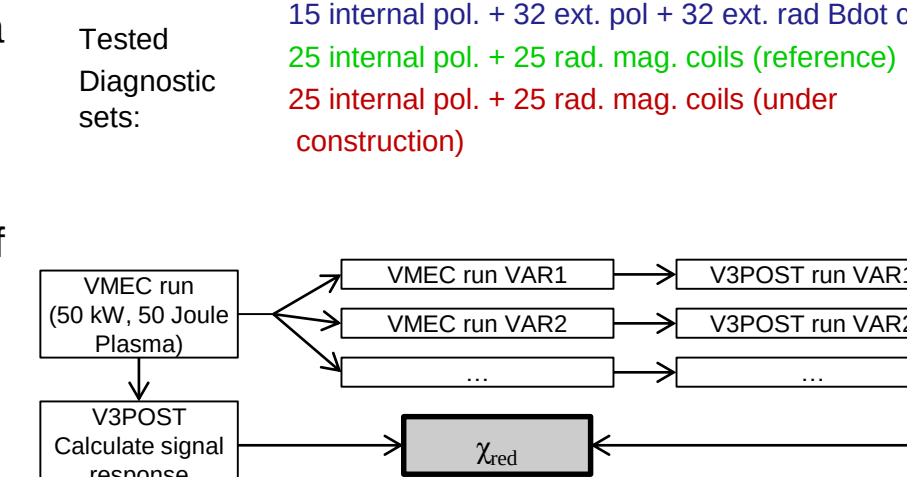
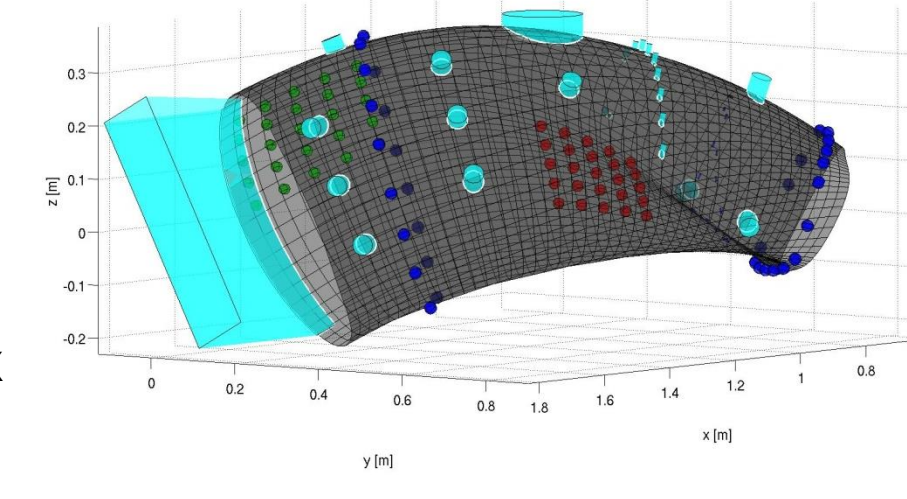
- (factor of 10 by using Thomson Scattering system in addition)
- Based on current/pressure profile an error profile is determined to allow an error estimation weighted by flux surface volume

$$\text{err} = \frac{E(p_{\text{err}}) - E(p_{\text{correct}})}{E(p_{\text{correct}})} \cdot 100\%$$

- Planned magnetic diagnostic set leads to suppression of solutions which strongly deviate from correct equilibria

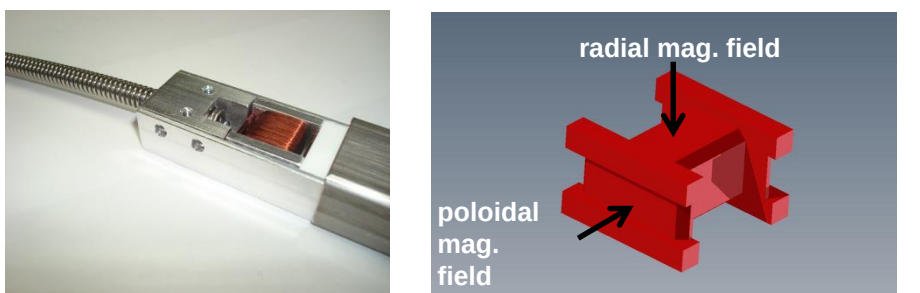
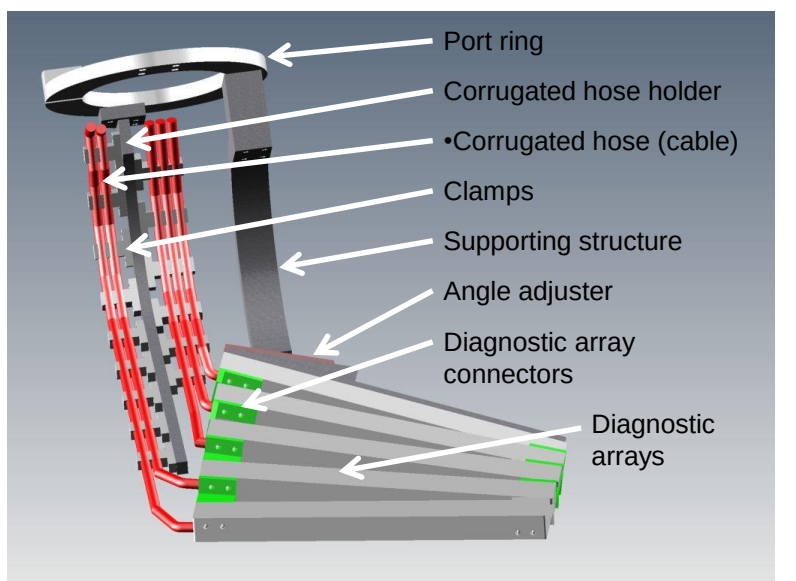


- Reducing sigma by two orders of magnitude allows significant reduction of number of solutions
- Further reduction can be achieved by including Thomson Scattering (TS) measurements for the plasma core (largest uncertainty in the reconstruction results)
- Weighting of one TS channel is adjusted to the 50 magnetic coil signals



## Diagnostic Construction

- Five arrays consisting of 5 poloidal and 5 radial magnetic coils, placed inside the vacuum vessel
- Same direction vector for all pol./rad. coils in one array
- Fixed distance between all coils allows the determination of the position using the measured magnetic field pattern
- Flat design to avoid touching the plasma
- Arrays, supporting structure, and corrugated hose holder follow HSX geometry
- Adjusted coil geometry for flat diagnostic array
- Ceramic used as coil body
- Stainless steel used as shielding for the coils
- Each array can be separately calibrated, replaced, and upgraded with more coils



## Summary

- Plasma equilibrium reconstruction at HSX is based on the determination of seven free parameters describing plasma current profile, pressure profile and plasma size
- Signal effectiveness has been used to find the most sensitive diagnostics
- Off-axis peaks in plasma current densities are challenging to reconstruct due to overlapping diagnostic positions with a high signal effectiveness
- Single parameter dependence study introduced as additional tool to find the final composition of diagnostics with high signal effectiveness
- Number of solutions will be greatly reduced with new set of magnetic diagnostics
- Error in remaining solutions is greatly reduced as well
- Diagnostic set is under construction, completion 12/2012
- New magnetic diagnostics allow benchmarking of different theories applied for positioning diagnostics and the effectiveness of single coils and subsets [5]

## References

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